How far is the Moon?

Use of high school geometry in calculating astronomical distances

Metadata

General Info

Title:
How far is the Moon?

Short description
Have you ever tried to see a small object alternately with one eye or the other? It seems to be moving, when in fact it is stable. This phenomenon is called parallax. Now think that you have a head the size of the Earth, so the one eye is at the northern hemisphere and the other eye is at the southern hemisphere and both of them are located at the same meridian. So if you observe the Moon alternately with the one eye or the other, you'll realize that the position of the Moon in the sky, is changing. This is the key to measure the distance to the Moon geometrically and we use the digital tool of Stellarium for that.

Keywords

Geometry, parallax angle, meridian, stellarium

Educational Context

Context

- 6th Grade of Primary School, Science, Modern Physics-Technology-Environment-Space, Solar System
- 2nd Grade of Junior High school, Mathematics, Trigonometry
- 2nd Grade of High school, Physics (orientation), Introduction, Types of Uncertainties
- Skills Laboratory

Age
15-18

Prerequisites

Basic trigonometry and geometry

Lv. Of difficulty

It depends on how many assumptions we'll take into account for the calculation of the distance between the Earth and the Moon.

Duration
(3 hrs max)

Educational Objective

Cognitive Objectives

- To solve an astronomical problem using mathematics

Affective

- To understand how useful the geometry really is in solving problems that students couldn't imagine.
- To value scientific thinking and efficiency

Psychomotor

- Cooperation among the students

Subject Domain

Big Ideas of Science

At the birth of Astronomy, Geometry was its vital base. The parallax angle can be used not only for the distance of close stars.

Subject Domain

Astronomy, Geometry

Orienting & Asking Questions

Orienting: Provide Contact with the content and/or provoke curiosity

Could a school student, like you, calculate the distance between the Earth and the Moon?

Credit: Apollo 8, NASA (1968)

Is the Euclidean geometry enough for this?
Take a look at the following video about basic geometry https://www.youtube.com/watch?v=wQAiytWGgEk

Define Goals and/or questions from current knowledge

Our goal is to understand the very limited accuracy we have and to estimate that, with only basic geometrical knowledge of junior high school and some assumptions, we can calculate astronomical distances in a very satisfactory level, which shows, at your eyes as students, that the scientific way of thinking is really working in all the aspects of life.

Take a look at the following video about parallax
https://www.youtube.com/watch?v=iwlMmJs1f5o

We'll follow the same pattern on geometry, but we'll change the use of it.

Hypothesis Generation and Design

Generation of Hypotheses or Preliminary Explanations

Let's say that your school is in the northern hemisphere and you'll observe the Moon simultaneously with the students of another school in the southern hemisphere and at the same meridian with you. We call meridian, the semicircle that goes from the North Pole to the South Pole on maps of the world.
Will you, both schools, observe the Moon at the same spot on the sky?
If there is a differentiation, could you depict it?

When we can’t find a collaboration for that project in order to have simultaneous observations, we could do the same work through an application, like Stellarium.

Design/Model

You can simulate the past or future sky, anywhere on Earth or even in other places in our Solar System through Stellarium. You can also toggle labels on and off, increase or decrease the number of visible objects, and many more.
Stellarium’s own YouTube channel contains several detailed videos showcasing the software’s capabilities, including how to remotely control a telescope:

https://www.youtube.com/channel/UC04hR2mrcRaM9MtMSG7uWLA.

Planning, Investigation and Analysis

Plan Investigation

We will use the method of parallax angle to measure the distance to the moon, as we told before.

We' ll set two locations: Cape town at the southern hemisphere and Stockholm at the northern hemisphere. These two locations are approximately at the same meridian. This fact is crucial, because through Stellarium we will measure the angular distance between the two positions from which the Moon is seen for the two selected locations, simultaneously.

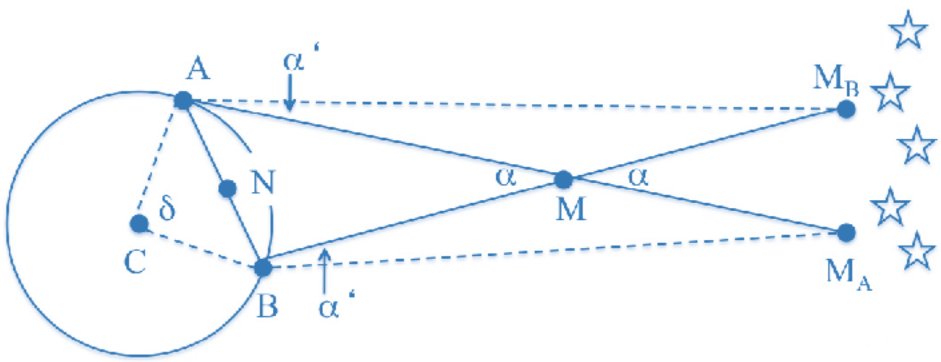


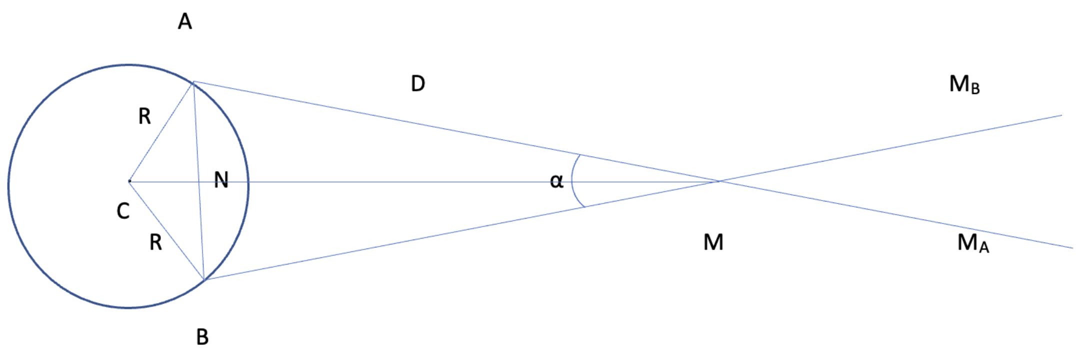
Image by Davide Cenadelli, Albino Carbognani, Andrea Bernagozzi and Cristina Olivotto

Think about from the figure above that point A is for Stockholm and point B is for Cape Town at the sphere of the Earth. Point M is for the Moon and the points M_B and M_A are the apparent positions of the Moon in front of the stars' background. Apparent position is the position that Moon seems to have on the sky.

But our whole demonstrator contains some assumptions in order to make the calculation possible for high school students without losing scientific logic.

Assumptions

a) We can assume that the ABM triangle is isosceles because the distance between the two locations and the Moon are approximately the same. With this assumption, we have to take into account the figure below for our calculations.
b) An assumption is that the Earth is completely round with a radius of 6,371 km.
c) Because the two locations are approximately at the same meridian, we also assume that the points A, B, C, N and M are at the same plane.
d) AM_B // BM_A



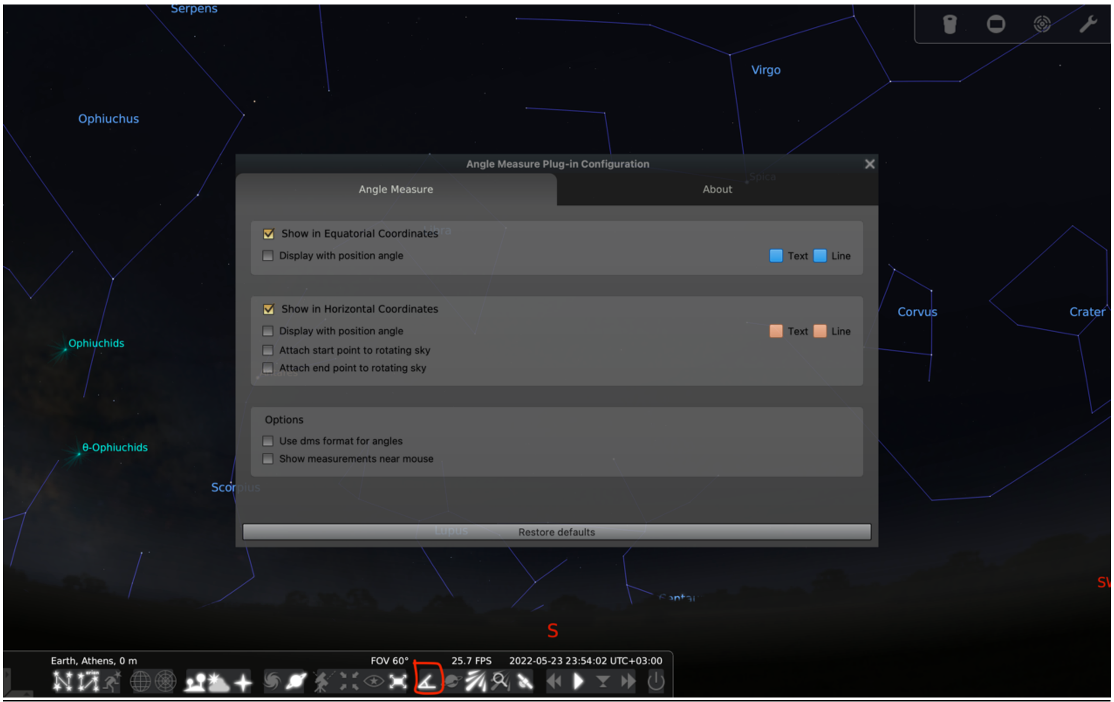
Selected location	Geog. Latitude (°)	Geog. Longitude (°)
Stockholm	59.3294 N	18.0687 E
Cape Town	33.9249 S	18.4241 E

The angle ACB is the sum of the absolute values of the two latitudes, so ACB = 93.25°.

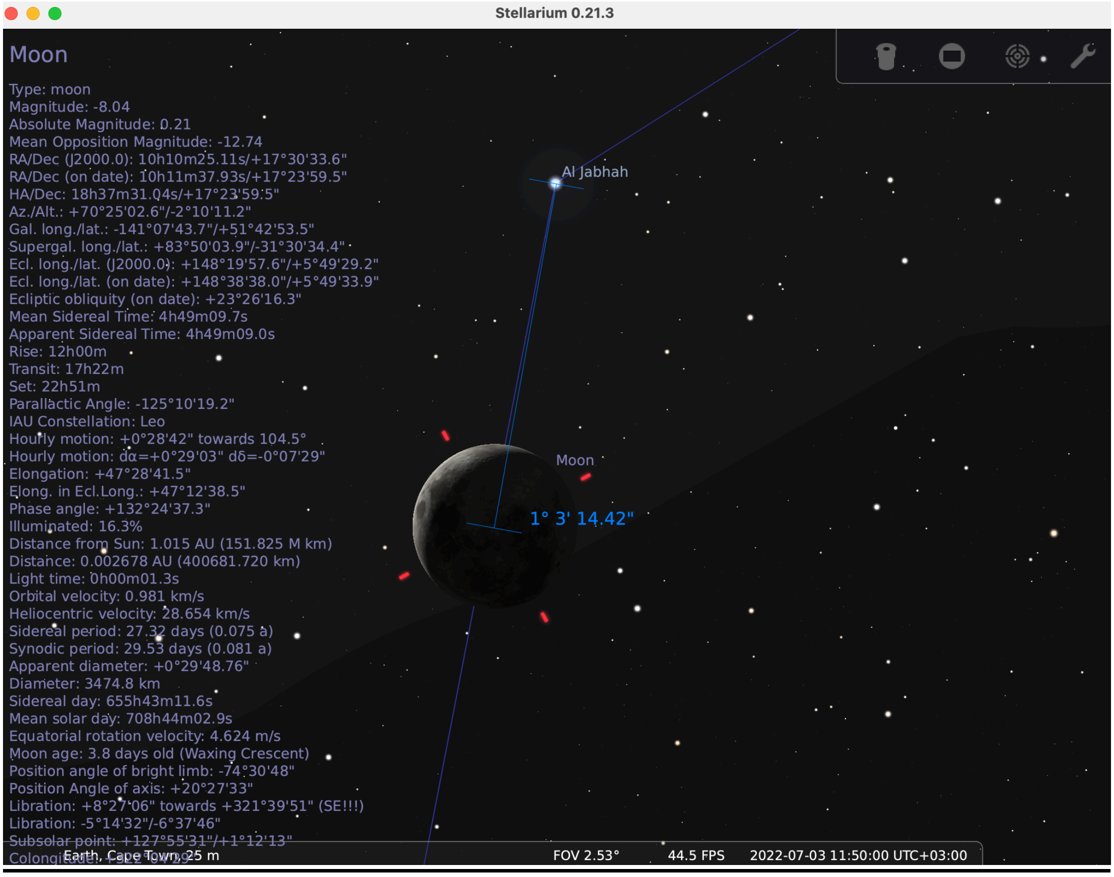
According to trigonometry, for the CAN triangle, sin (ACN) = AN/AC and you can find AN.

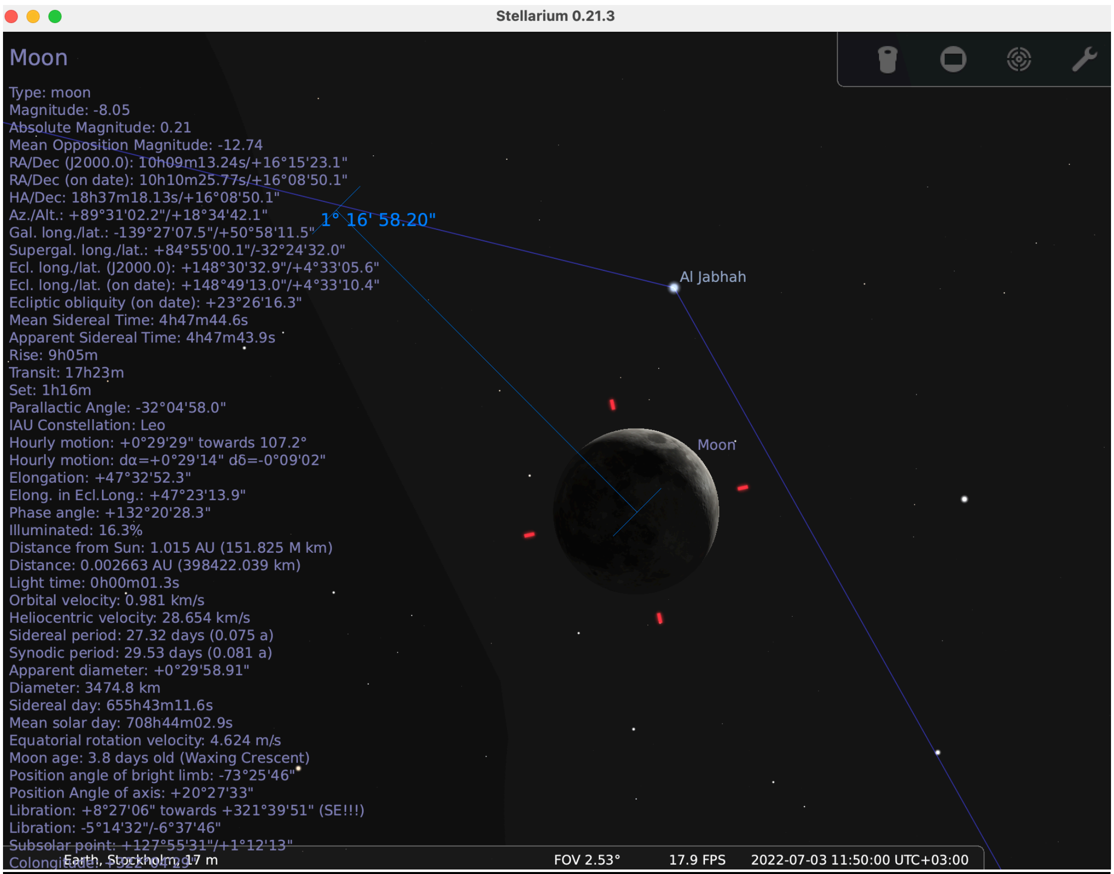
Now for the NAM triangle, sin $(\frac{\alpha}{2})$ = AN/ D.

D is the unknown quantity we calculate for this demonstrator, because is the distance between one of the selected locations and the Moon, α is the angular distance between the two apparent positions of the Moon (the sky is like a dome and we measure the relative distances to degrees) that we measure through Stellarium with the red outlined button that seems at the figure below. From the plug in, you may add some additional tools like this one that is called “angle measure”.



At the following images, you can see the Moon simultaneously from the two different locations. Firstly, we measure the angular distance between the center of the lunar disc and a close star to it, for the first location. Secondly, we take advantage of that Stellarium keeps the previous measurement and we set the right click of the mouse to drag for the measurement of the angular distance between the centers of lunar discs (from the first and the second observation).





An example of angle α measurement

The time that we have selected for the calculation is showed on the figures. Try to find a moment for observation that Moon is approximately in the same altitude for both locations.

For the conversion and the calculation of sinus, use wolfram alpha https://www.wolframalpha.com/ which is a very easy and direct online tool for a huge range of calculations.

Analysis & Interpretation

Analysis and interpretation : Gather result from data

So, what is your calculation? The average distance to the moon is 382,500 kilometers. Is your calculation close enough to it?
Additionally, you can calculate the percentage error of your "observation" from the following formula and if it is lower than approximately 5%, your calculation is really very accurate!!

$$\%\ error = \left(\frac{accepted - observation}{accepted}\right) \times 100$$

If it is higher than 5%, never mind, because we've taken many assumptions into account and the mouse drag for measuring the angular distance is not very accurate.

Conclusion & Evaluation

Conclude and communicate result/explanation

Let's think about according to the results.
What does the accuracy of your results depend on?
How long does the reflected light take to travel from the Moon to the Earth?
How long does it take for the Orion spacecraft of the ARTEMIS I mission to reach the Moon if it has a speed of 26400 mph or 39580 kph?

Now you have to realize how far is our closest celestial neighbor!!

Evaluation/Reflection

Maybe every team can present its results and the whole demonstrator would become a pleasant geometrical contest with poster from every team!

Of course, each of your teams will take different locations with the same criteria to solve the demonstrator and come to the expected result.

About the importance of this project for you, rate from 1 (lowest) to 5 (highest) each topic:

- The interest that this activity provoked to you
- The desire to talk your classmates about it
- The knowledge acquired/reinforced
- The skills you obtained